Kinematics of a particle

Tutorial 3

Normal-tangential co-ordinates

Question 3.1

A flywheel $2.4m$ in diameter accelerates at a steady rate from rest to $3500\ rpm$ in 30 seconds.

Determine:

a) Its angular acceleration.

b) The amount of revolutions the flywheel makes when it reaches $3500\ rpm$.

c) The linear velocity and the magnitude of acceleration of a point on the rim of the flywheel 5.8 seconds after it has started rotating from rest.

Question 3.2

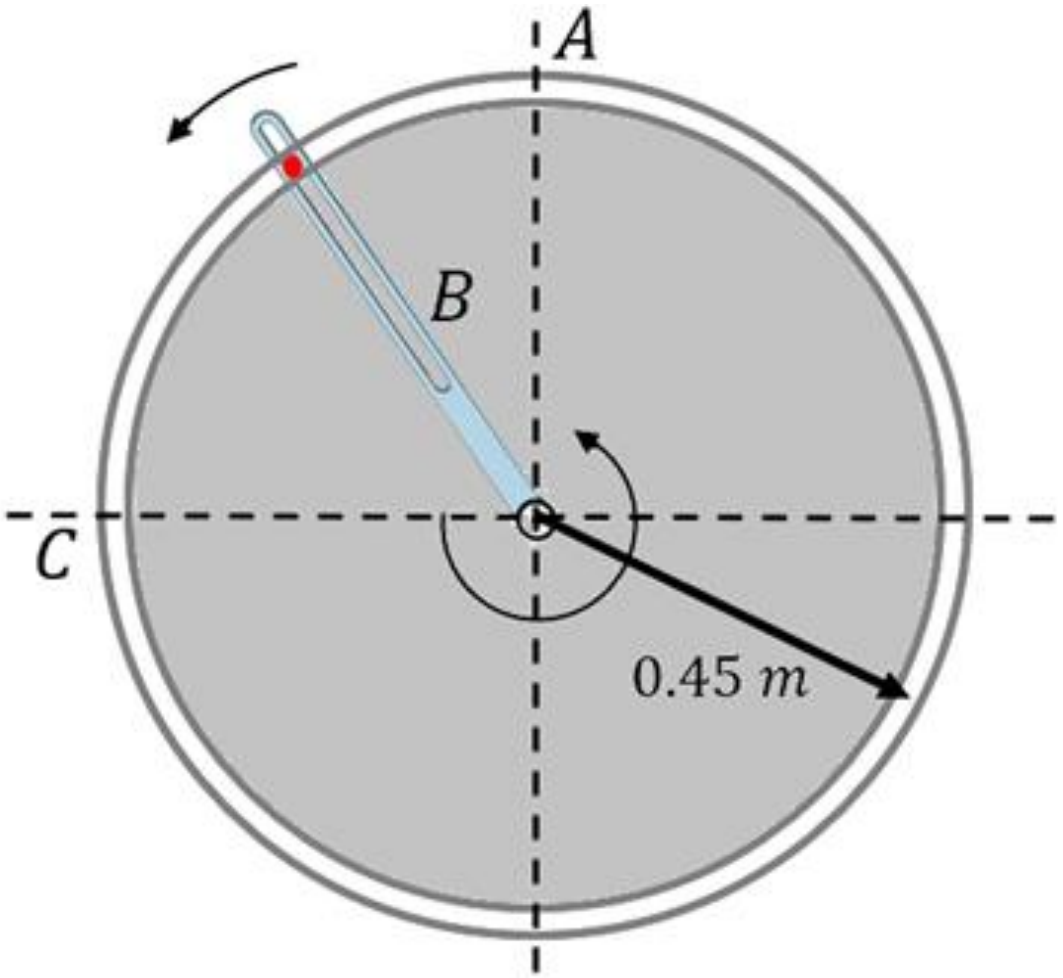
A small ball travels around the circular slot at a speed of 12 m/s by the arm B rotating about the centre. The speed of the ball begins to increase when it reaches point A, which is given by the equation: $a_t = \frac{2}{3}v^{0.75}$ m/s^2. Determine:

a) The equation of linear velocity v of the ball in terms of time t.

b) The equation of the distance s travelled by the ball in terms of time t.

c) The velocity of the ball at point C.

d) The magnitude of acceleration of the ball when it reaches point C.



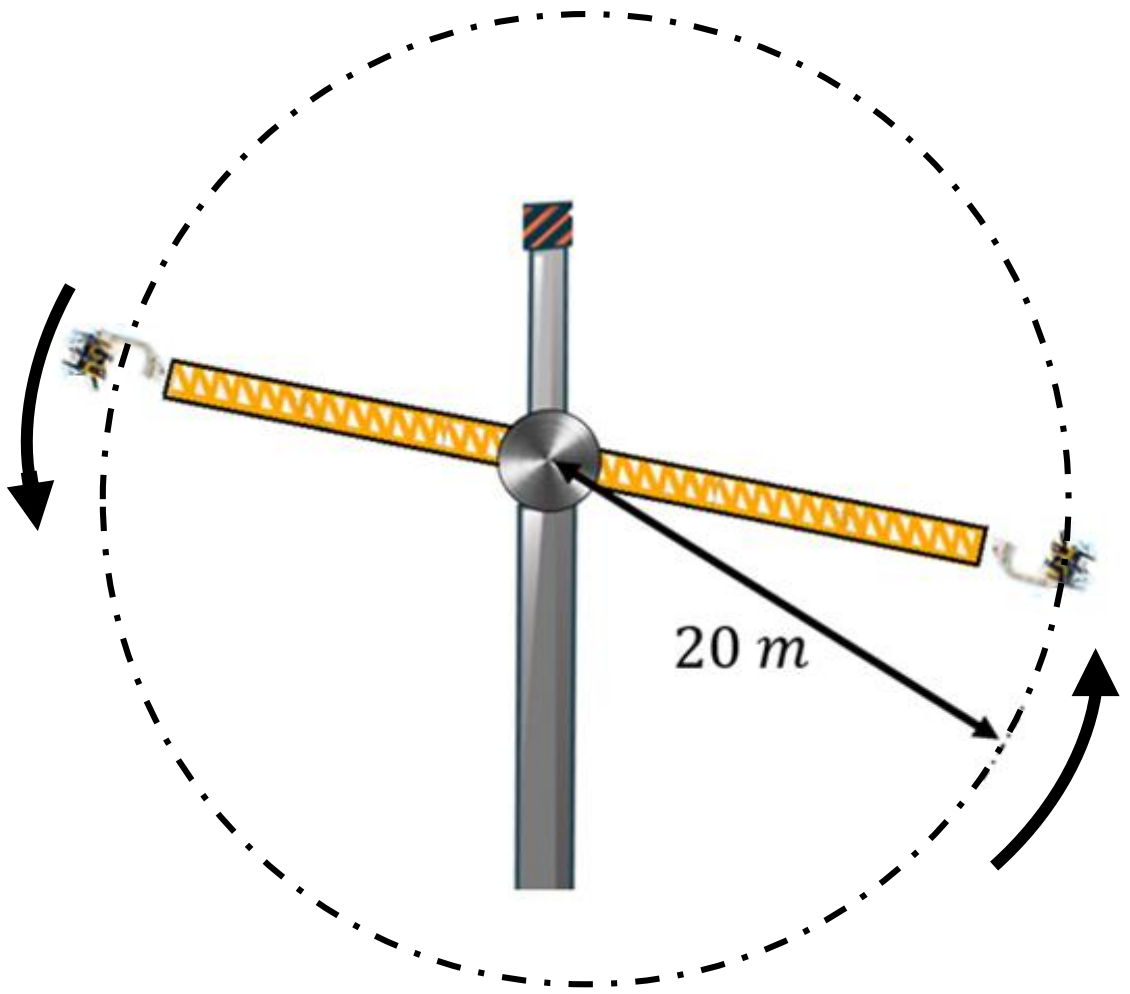
Question 3.3

An amusement ride is rotating around its centre giving the passengers a constant linear speed of $15\ m/s$. What is the magnitude of the passenger's acceleration a as they pass through:

a) The lowest point on their circular motion.

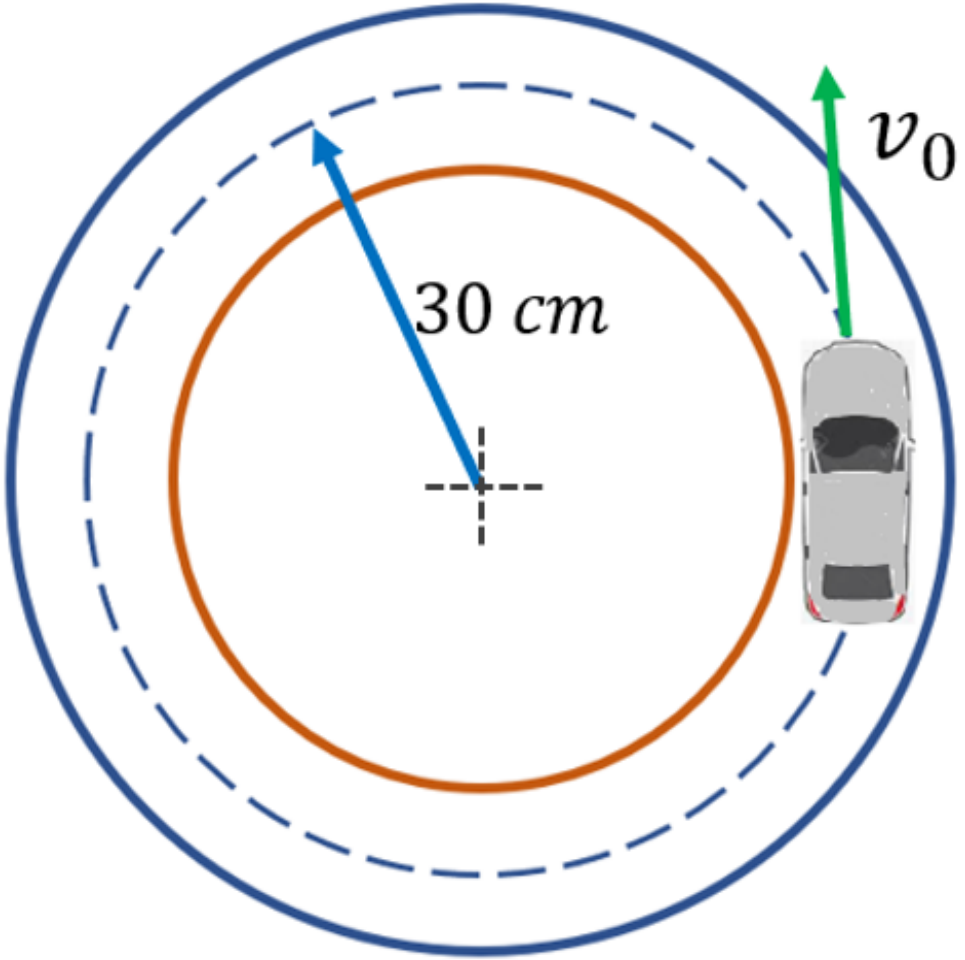
b) The highest point on their circular motion.

c) Find how long it takes for the passengers to complete one full revolution.



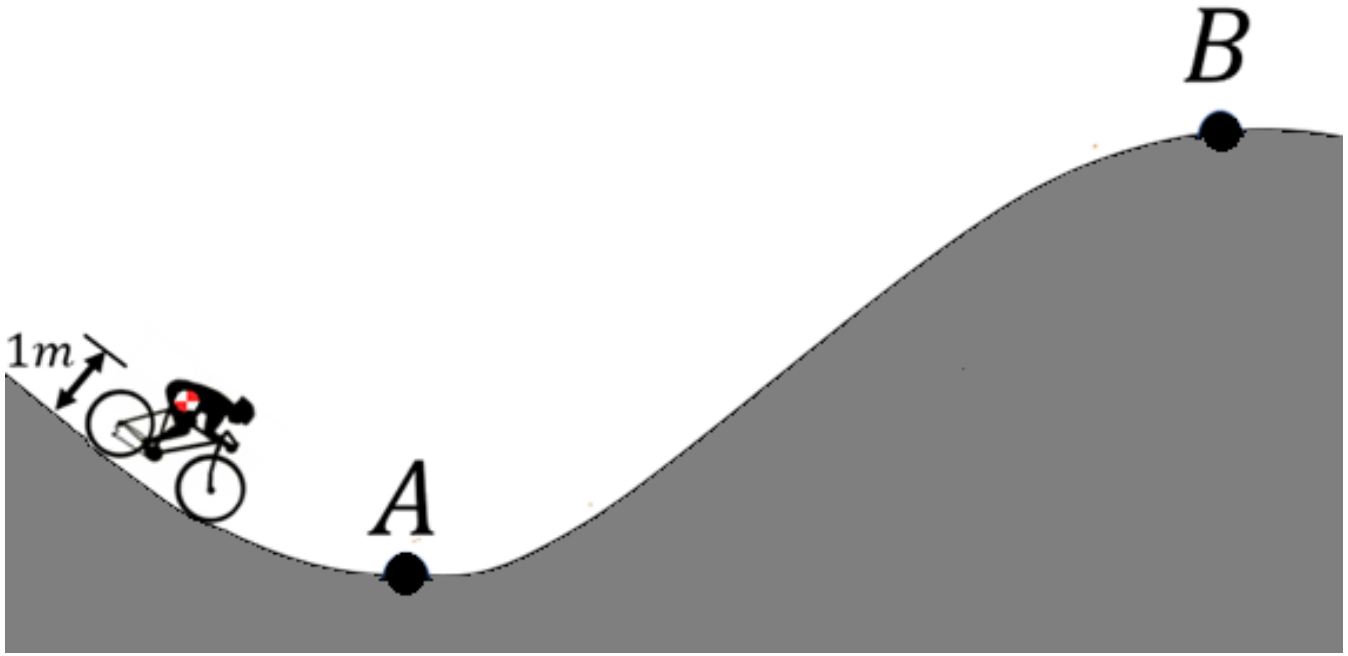
Question 3.4

Starting from rest, an electric model car is speeding up along a circular path at a steady rate reaching $5\ m/s$ in 4 seconds. Find the magnitude of the acceleration a 2 seconds after it starts motion.



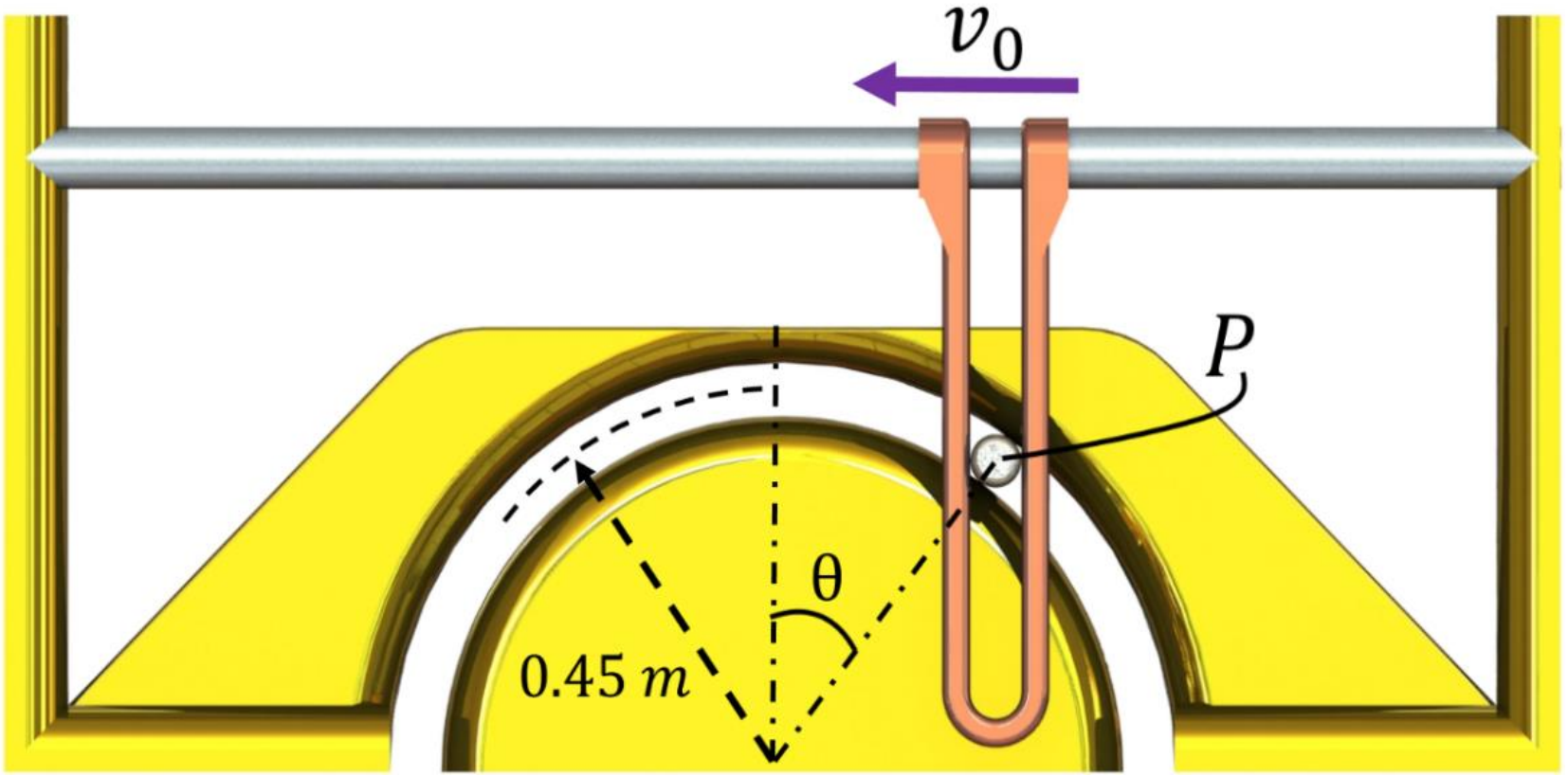
Question 3.5

A cyclist travels from the bottom of the track at point A to the top at point B at a constant speed. If the radius of curvature of the track at point A is $\rho_A = 50m$ and the acceleration is $0.55g$ what is the speed of the cyclist v_A at point A? If the acceleration at point B cannot exceed $0.35g$ what is the minimum allowable radius of curvature ρ_B of the track at point B?



Question 3.6

As shown in the simple mechanism below, the pin P in the fixed circular slot is being moved by a slotted guide with a constant horizontal velocity $v_0 = 5\ m/s$. Calculate the normal and tangential components of acceleration a_n and a_t of pin P as it goes past the position where $\theta = 40^0$.

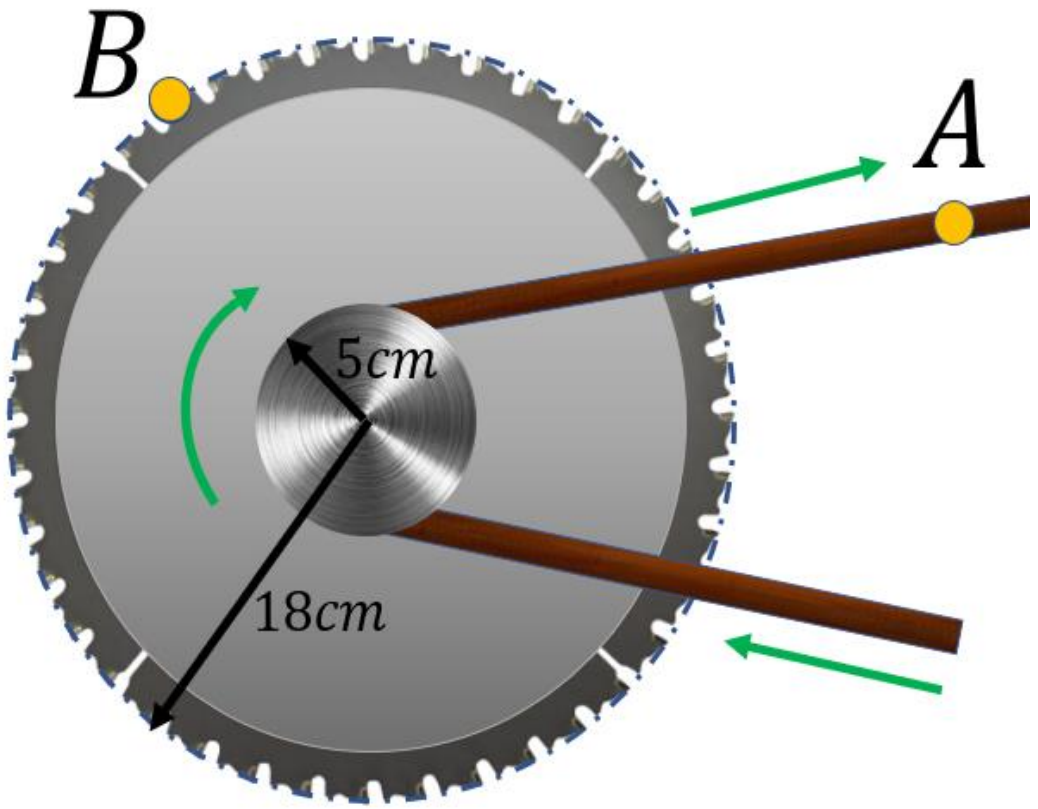


Question 3.7

A circular saw blade and attached pulley rotate around a fixed central shaft and are both driven by the belt. At an instant, the velocity and acceleration of the point A on the belt is $10\ m/s$ and $6.5\ m/s^2$ respectively. Calculate:

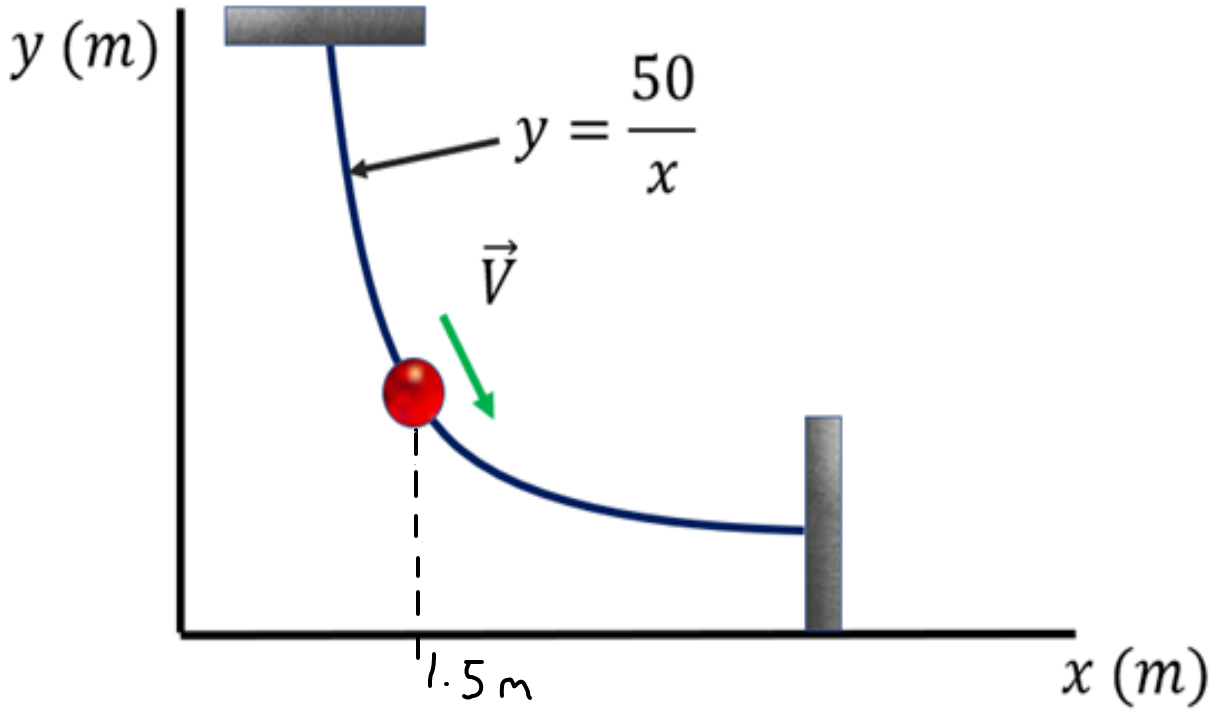
a) The normal and tangential components of acceleration at point B on the saw blade at this particular instant.

b) The magnitude of the total acceleration at point B on the saw blade at this particular instant.



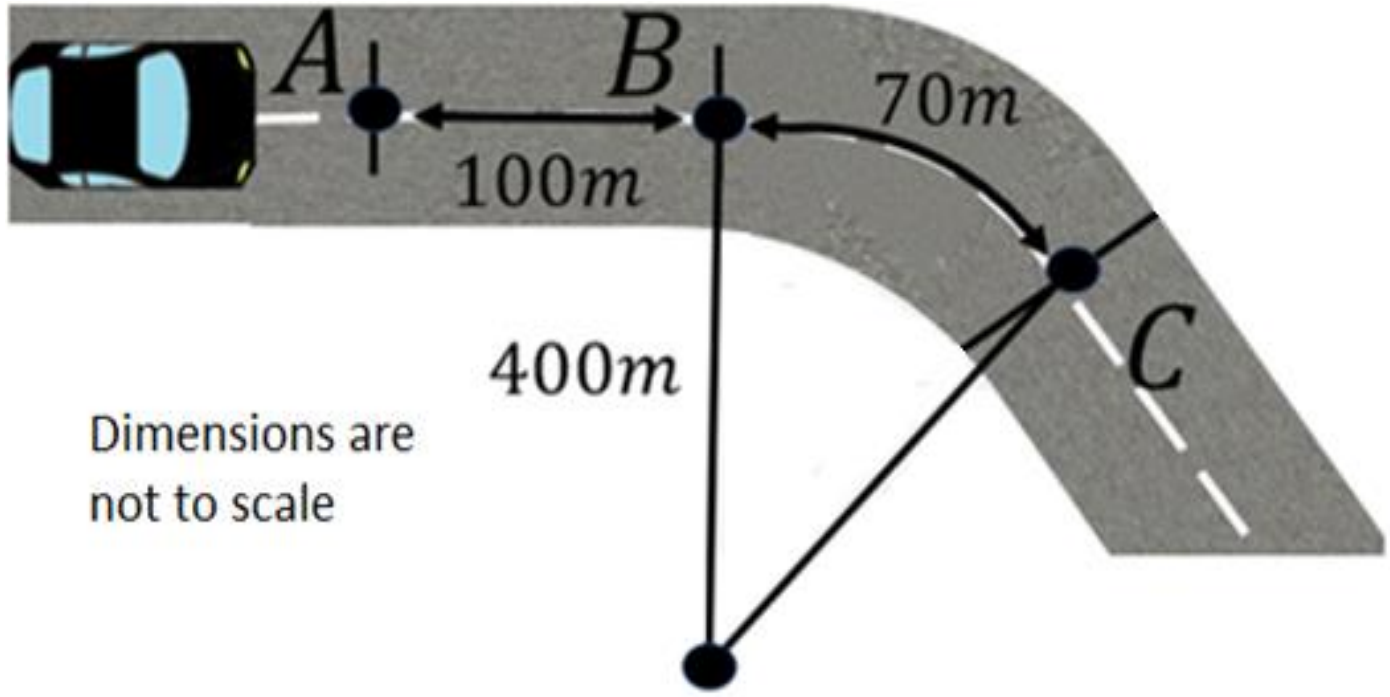
Question 3.8

A particle travels along the curve shown in the figure with a constant speed of $v = 5\ m/s$. What is the particle's acceleration when it is located at the point $1.5m$.



Question 3.9

A car is travelling at $25\ m/s$ in the straight part of the road. When the car reaches point A the brakes are applied and the car slows down at a steady rate to $15\ m/s$ when it reaches point C. What is the magnitude of the total acceleration of the car just after it passes point B?



Question 3.10

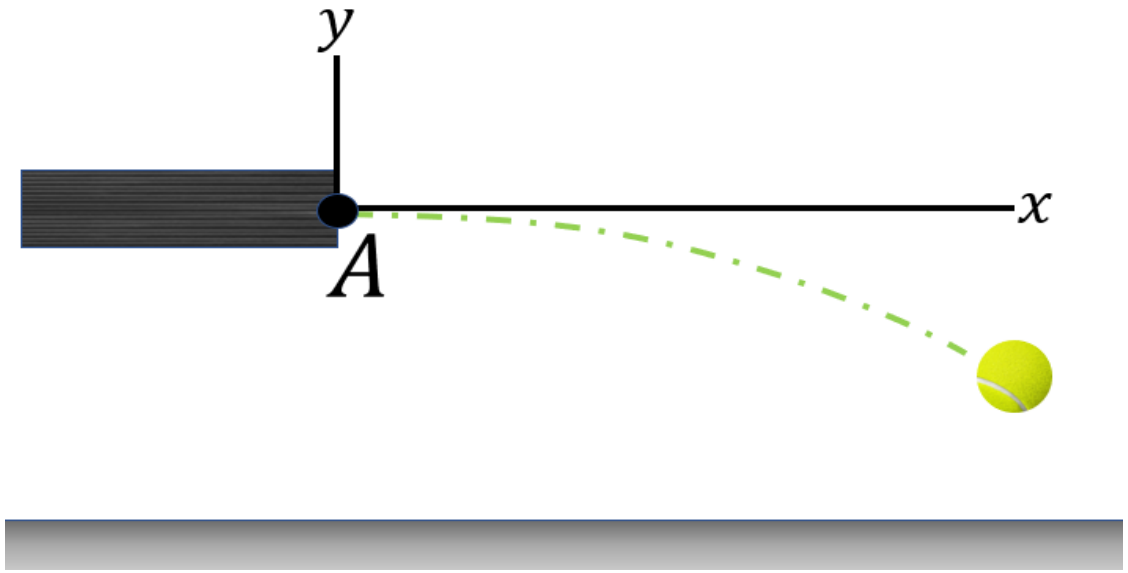
A tennis ball is shot out horizontally at A with a speed of $40\ m/s$.

Find:

a) The equation of the path ($y\ as\ a\ function\ of\ x$)

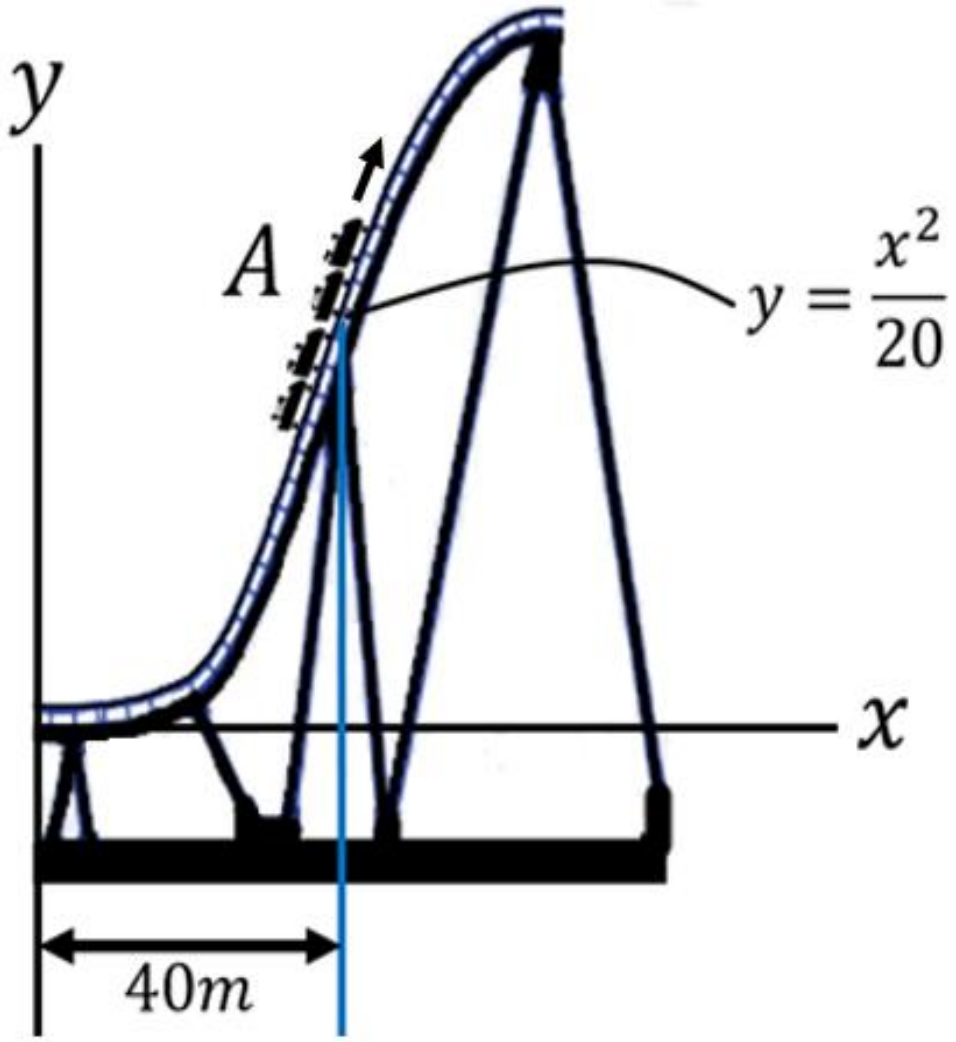
b) The tennis ball's velocity when time $t = 0.5\ s$.

c) The normal and tangential components of its acceleration when time $t = 0.5\ s$.



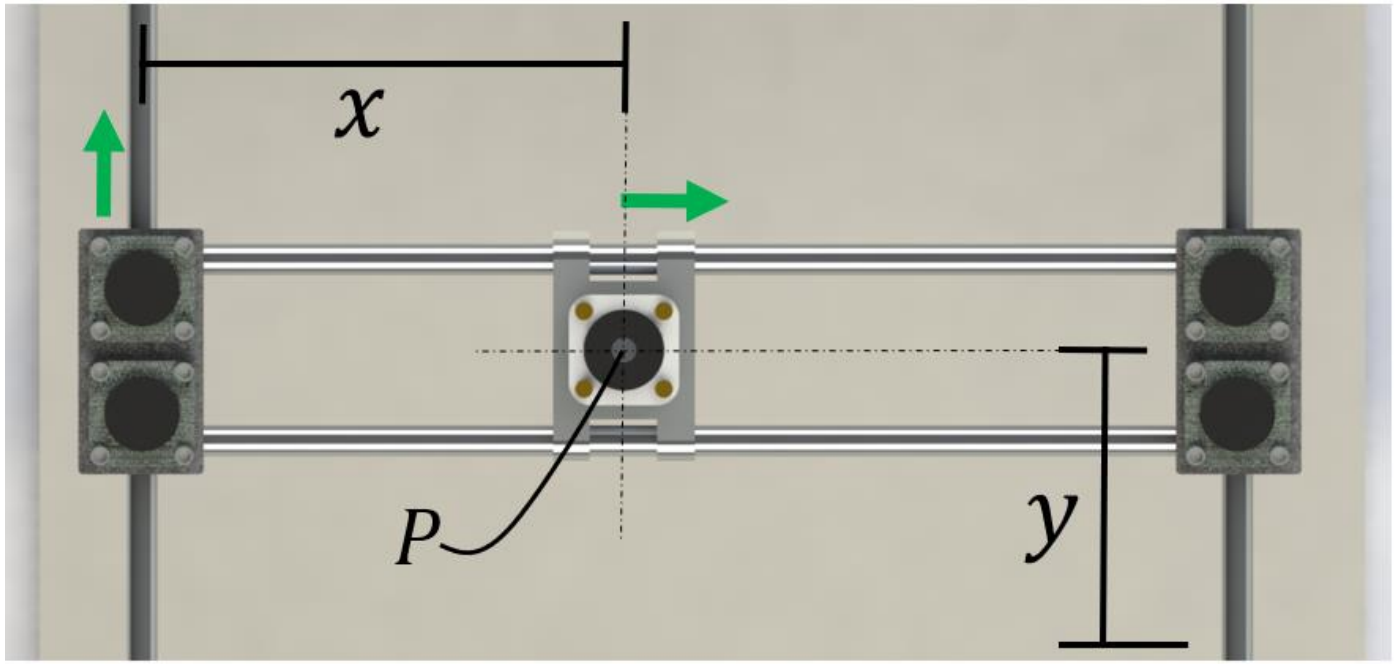
Question 3.11

If the speed of the rollercoaster at point A is $35\ m/s$ and is increasing at a steady rate of $9\ m/s^2$ in the tangential direction, calculate the magnitude of the total acceleration of the roller coaster at this instant.



Question 3.12

The mechanism of a $3D$ printer is designed to move the extruder at point P according to $x = 10 - 4t + 2t^2$ and $y = 4 + 20t - 8t^2$ for a short while, where x and y are in cm and time t is in seconds. Find the radius of curvature ρ of the path of the extruder at the instant where time $t = 1.12\ s$.



Answers are shown on the next page

Answers

Question 3.1

$\ddot{\theta} = 12.22\ rads/s^2$

$875.2\ revolutions$

$v = 9.46\ m/s$

$|a| = 76\ m/s^2$

Question 3.2

$$v = \frac{t^4}{1296} + 12$$

$$s = \frac{t^5}{6480} + 12t + s_0$$

$v = 12.25 m/s$

$|a| = 333.5\ m/s^2$

Question 3.3

$a_n = 11.25\ m/s^2$ Acting upwards towards centre

$a_n = 11.25\ m/s^2$ Acting Downwards towards centre

$T = 8.37\ s$

Question 3.4

$|a| = 20.87\ m/s^2$

Question 3.5

$v_A = 16.27\ m/s$

$\rho_B = 76.14\ m$

Question 3.6

$a_t = -79.44\ m/s^2$

$a_n = 94.67\ m/s^2$

Question 3.7

$|a| = 7200.74\ m/s^2$

Question 3.8

$|a| = 0.067\ m/s^2$

Question 3.9

$|a| = 1.53\ m/s^2$

Question 3.10

$y = -0.0031x^2$

$v = 40.3\ m/s$

$a_t = 1.194\ m/s^2$

Question 3.4

$|a| = 20.87\ m/s^2$

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$v_A = 16.27\ m/s$

$\rho_B = 77.14\ m$

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$a_n = 9.74\ m/s^2$

Question 3.11

$|a| = 3.028\ m/s^2$

Question 3.12

$\rho = 0.6\ cm$