Kinematics of a particle Tutorial 4

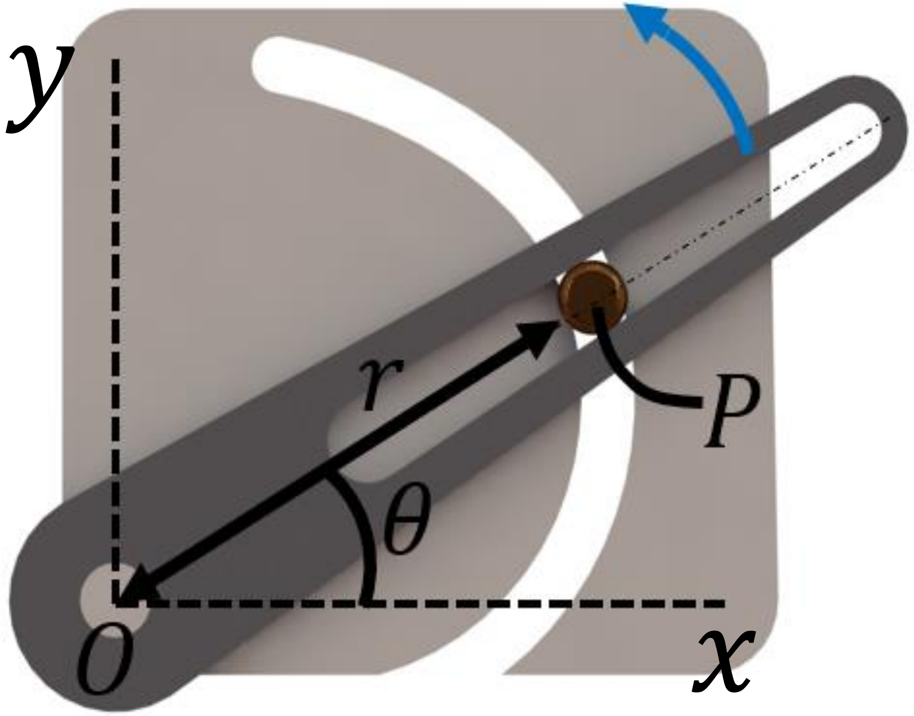
Polar, cylindrical and spherical co-ordinates

Question 4.1

A ball bearing P is driven by a slotted link which is fixed to rotate around the point O. The slotted link has a constant angular velocity $\dot{\theta} = 5\,\mathrm{rads/s}$ and drives the ball bearing P along the guided path which varies with the angular position θ according to $r = 2.5\theta$ where r is in meters and θ is in radians. Find:

a) The radial and transverse components of velocity v and acceleration a of the ball bearing P at the instant that $\theta = \frac{\pi}{6}$ radians.

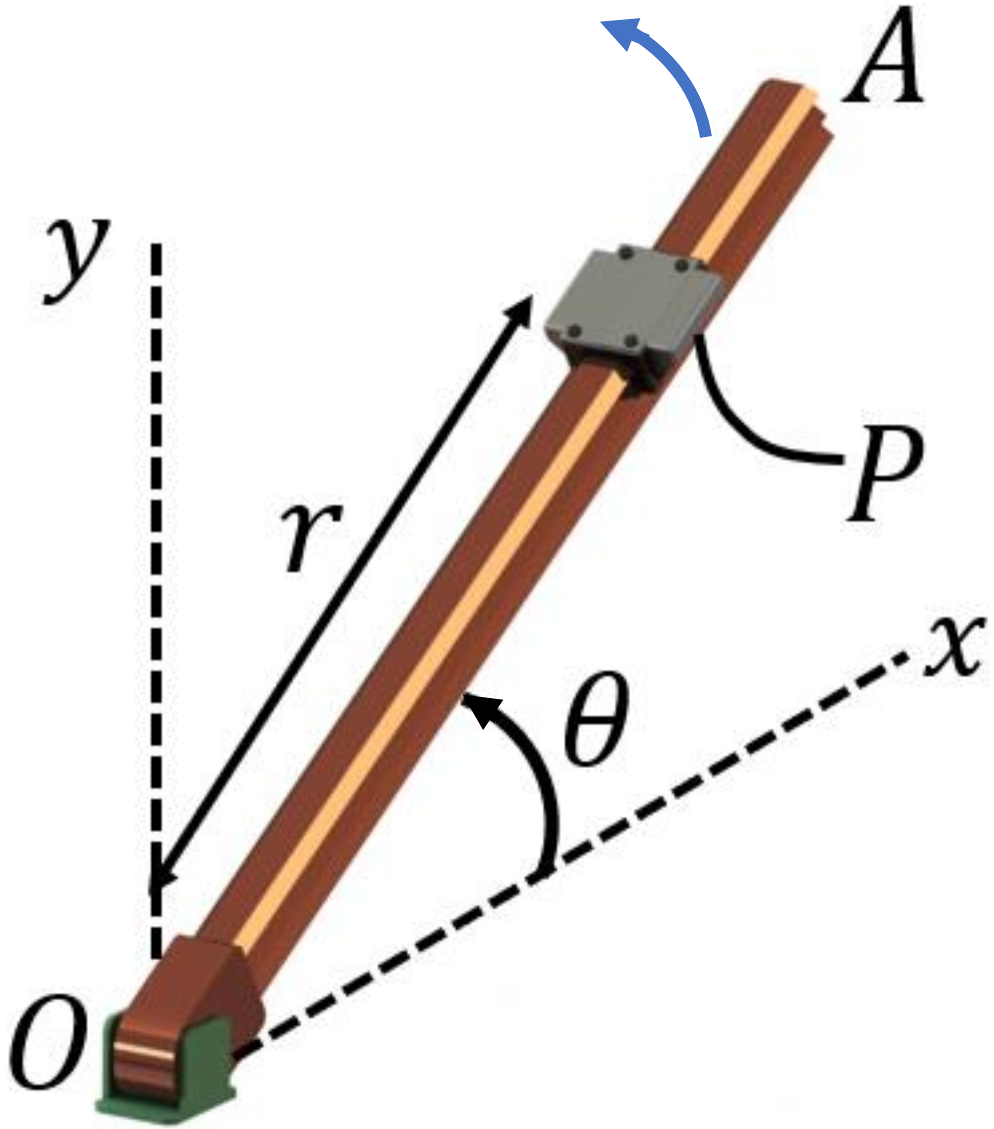
b) The magnitudes of velocity v and acceleration a of the ball bearing P when $r = 1.5$ m.



Question 4.2

PEM tutoring

A motorized slider P is moving towards the point O along the rail OA. At the same time the rail can rotate about the point O according to $\theta = 0.4t^4 + 0.2t^2 + 0.25t$ where θ is in radians and t is in seconds. The position of the slider P along the rail follows $r = \ 0.02t^3 + 0.7t^2$.



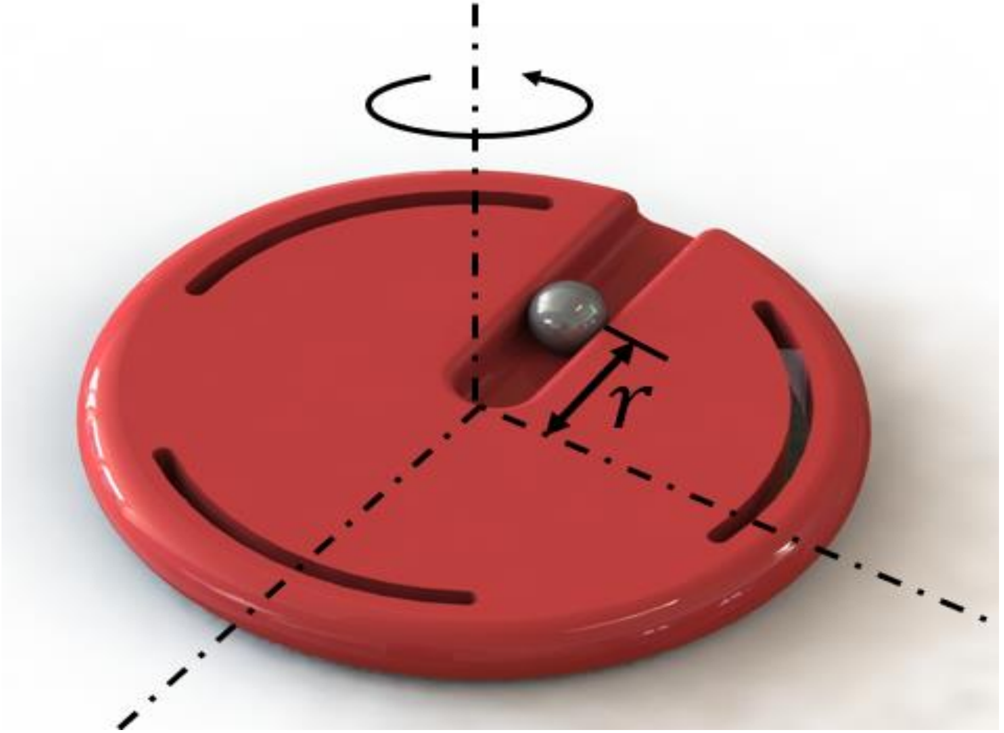
Determine:

a) The velocity v and acceleration a of the slider at time $t = 2\text{ s}$.

b) The angles v and a make with the positive x axis.

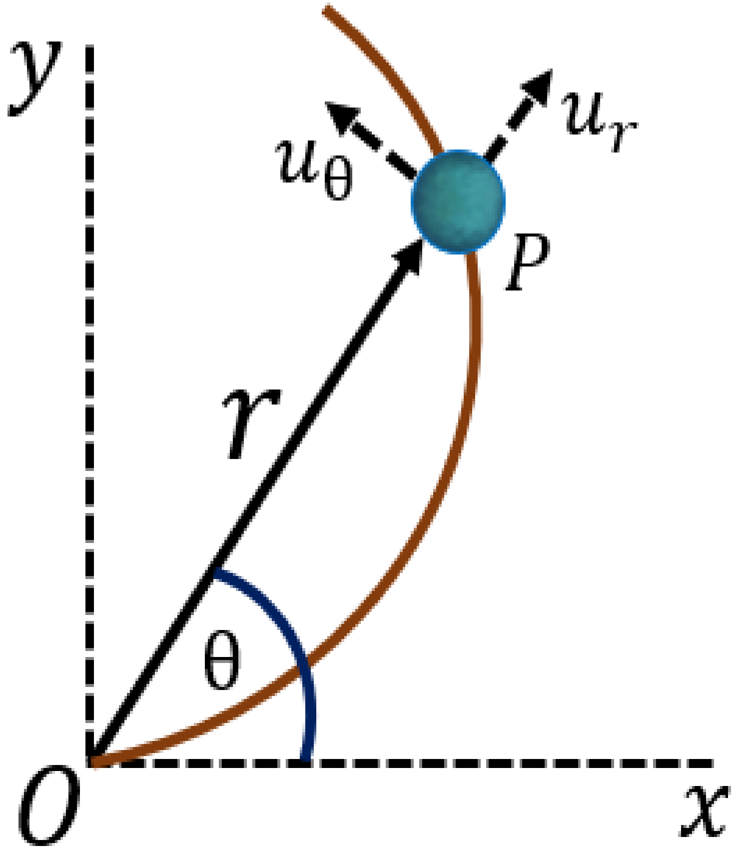
Question 4.3

A ball bearing moves outwards along the slot on a rotating disc with an acceleration of $\ddot{r} = 2t$ where t is in seconds. The disc has a constant angular velocity of $8\,\mathrm{rads/s}$. If the ball bearing starts moving from the centre from rest, determine what the magnitudes of velocity v and acceleration a would be at the instant the ball bearing leaves the disc at time $t = 1.85$ s.



Question 4.4

The particle P moves along the path described by the equation $r = \pi\theta$ m. If the angle varies with time t according to $\theta = t^3$ radians, find the velocity and acceleration of the particle when $\theta = 40^0$. **Use two methods.**



Question 4.5

A particle P moves along a spiral path $r = 5\theta$, if the particle moves at a constant angular velocity of $\dot{\theta} = 9\,\text{rads/s}$.

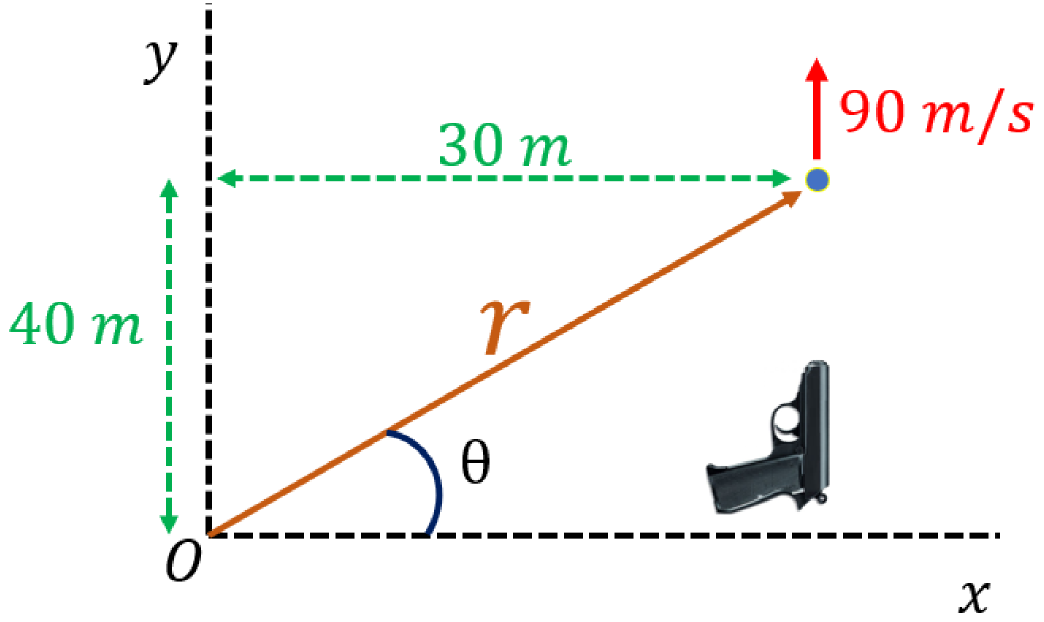
Find:

a) The radial and transverse components of velocity v and acceleration a of particle P when $\theta = \frac{\pi}{6}\text{rads}$.

b) The radial and transverse components of velocity v and acceleration a of particle P when there is now an angular acceleration of $\ddot{\theta} = 3\ \text{rads/s}^2$ at the instant $\theta = \frac{\pi}{6}\ \text{rads}$.

Question 4.6

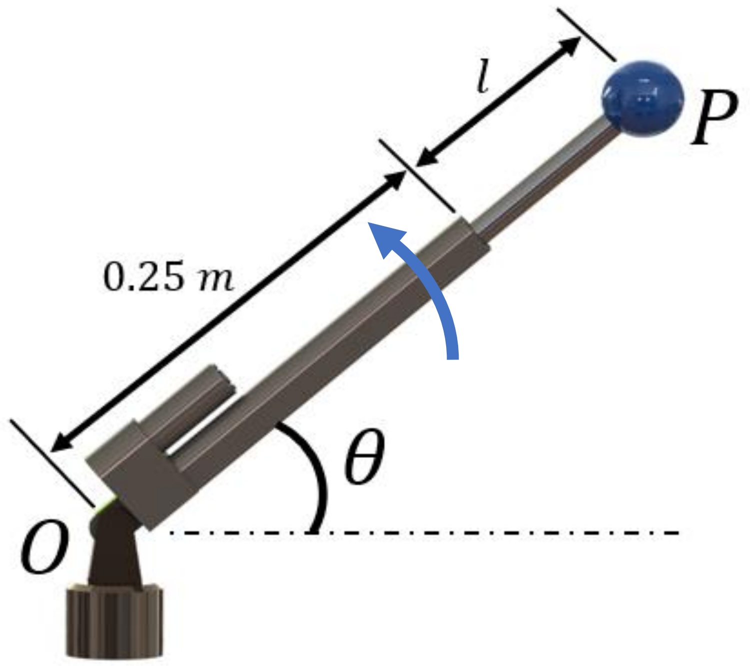
A ball bearing gun is shot vertically upwards with an acceleration in the y direction given as $a_y = -g - kv^2$ where the term kv^2 accounts for aerodynamic drag on the ball bearing with the constant $k = 0.015\ \mathrm{m}^{-1}$ and $g = 9.81\ \mathrm{m/s^2}$. If the velocity of the ball bearing at the instant shown is $90\,\mathrm{m/s}$ in the y direction find the radial and transverse components of its velocity v and acceleration a as seen from point O.



Question 4.7

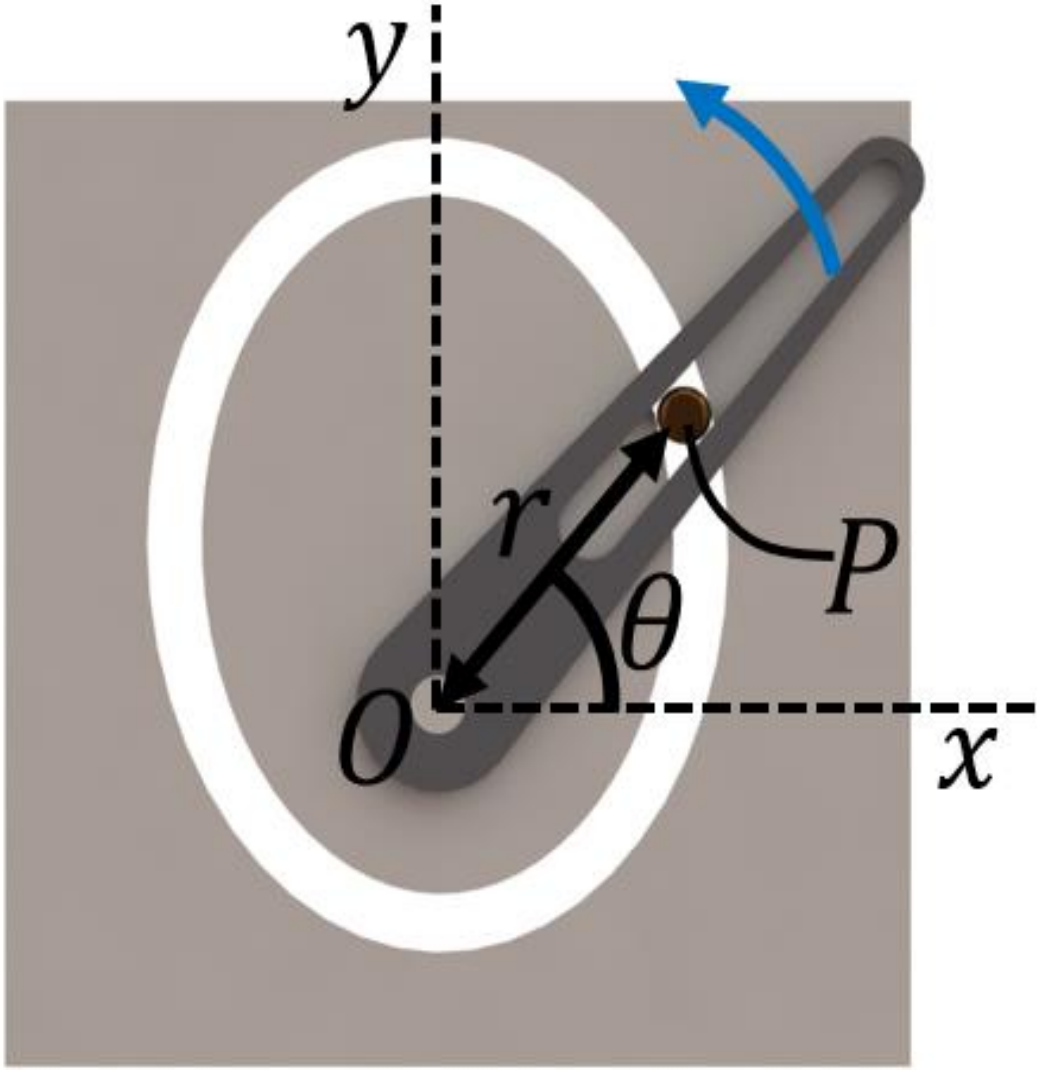
PEM tutoring

A linear actuator rotates around O, the length l is controlled according to $l = 0.2\sin\frac{\pi t}{3}$ where l is in meters and t is in seconds. At the same time the linear actuator is programmed to rotate according to $\theta = \frac{\pi}{2} + 0.8\sin\frac{\pi t}{3}$ radians. Find the magnitude of velocity v and acceleration a of the point P on the end of the length l at the instant where time $t = 4$ s.



Question 4.8

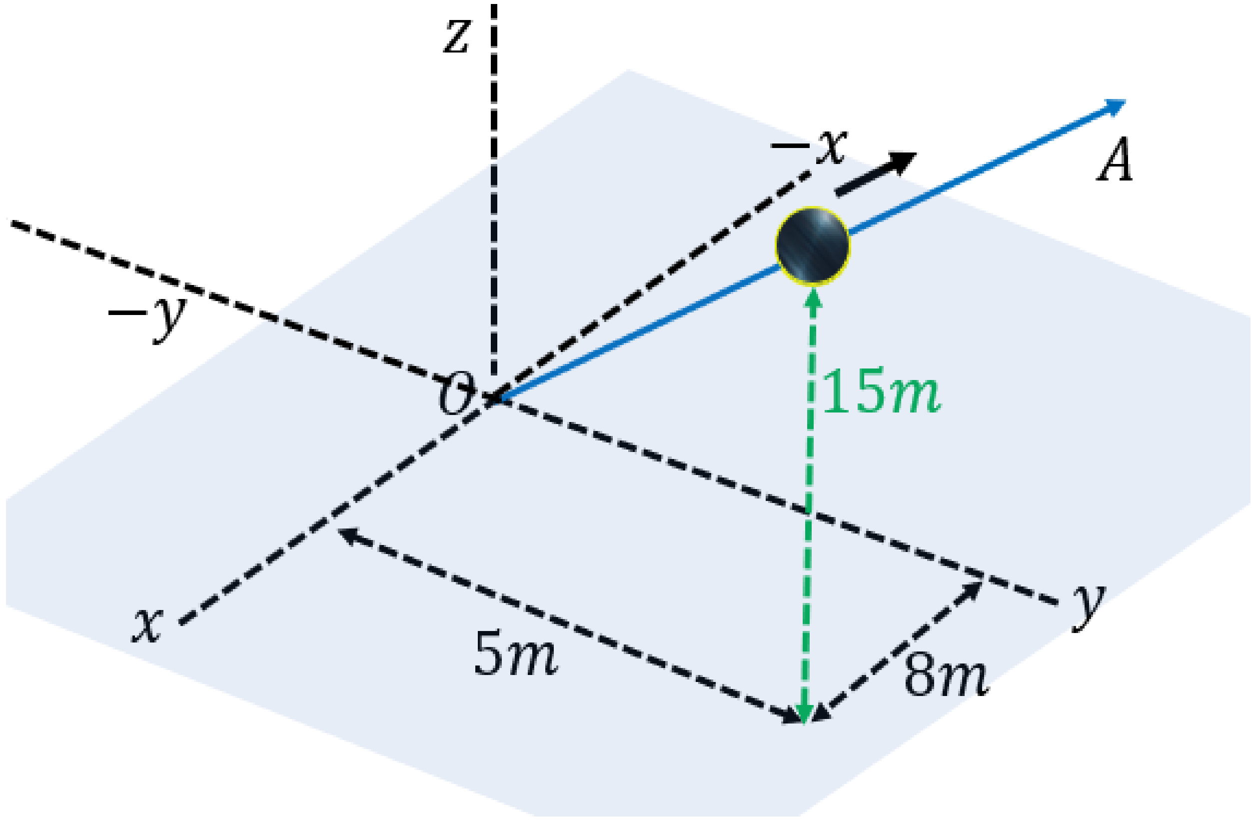
The ball bearing P driven by the slotted link in question 4.1 now follows a new guided path which varies with the angular position θ according to $r = \sin\theta + 0.5$ where r is in meters and θ is in radians. If the slotted link has a constant angular velocity $\dot{\theta} = 10\ \text{rads/s}$ what would be the maximum speed of the ball bearing?



Question 4.9

PEM tutoring

A particle moves along the straight length OA. At the instant shown its speed is $8\ \mathrm{m/s}$ and has an acceleration of $2\ \mathrm{m/s^2}$. Write the velocity and acceleration of the particle in terms of cylindrical components.

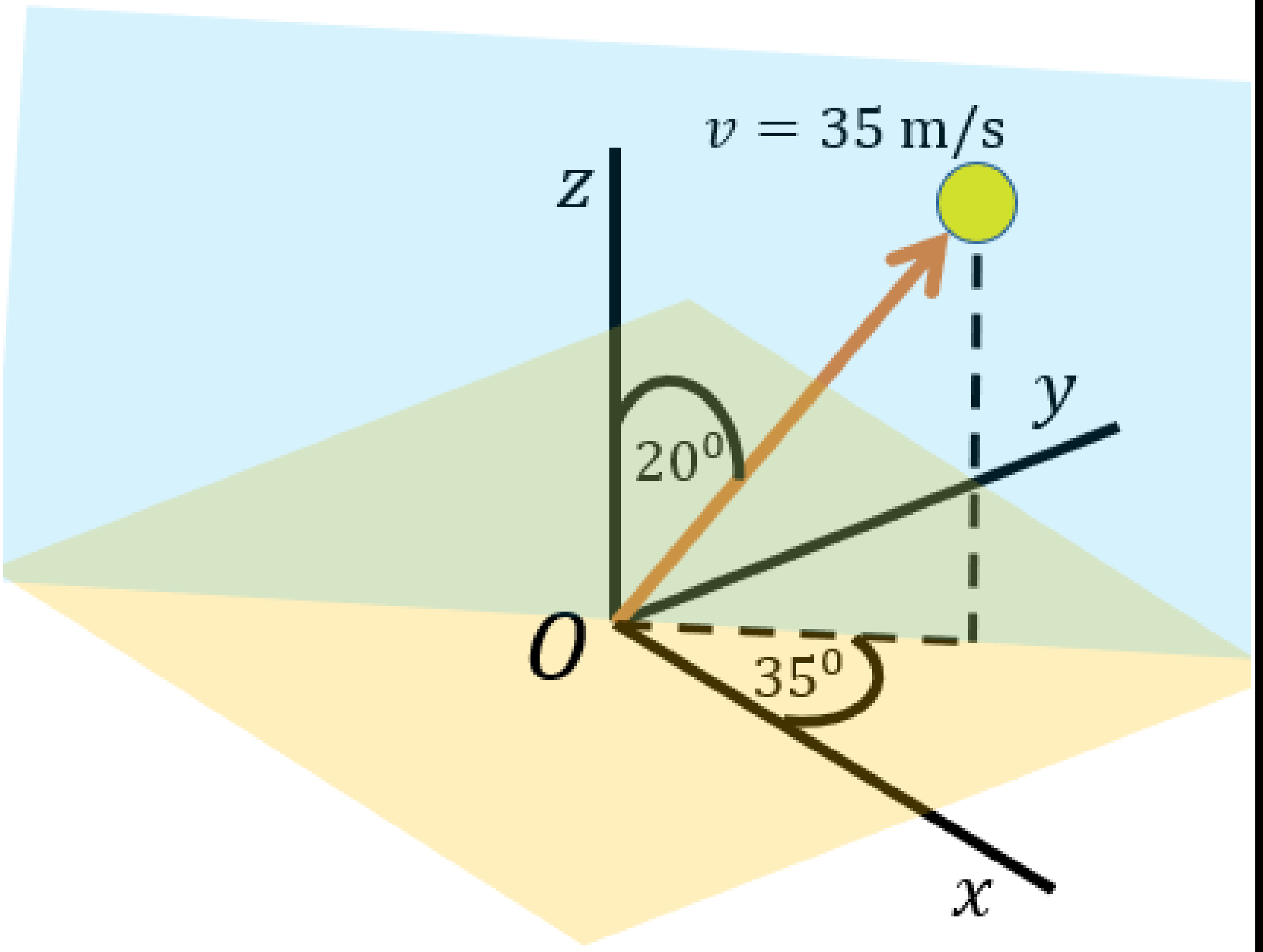


a) Using trigonometry

b) Using the vector dot product method.

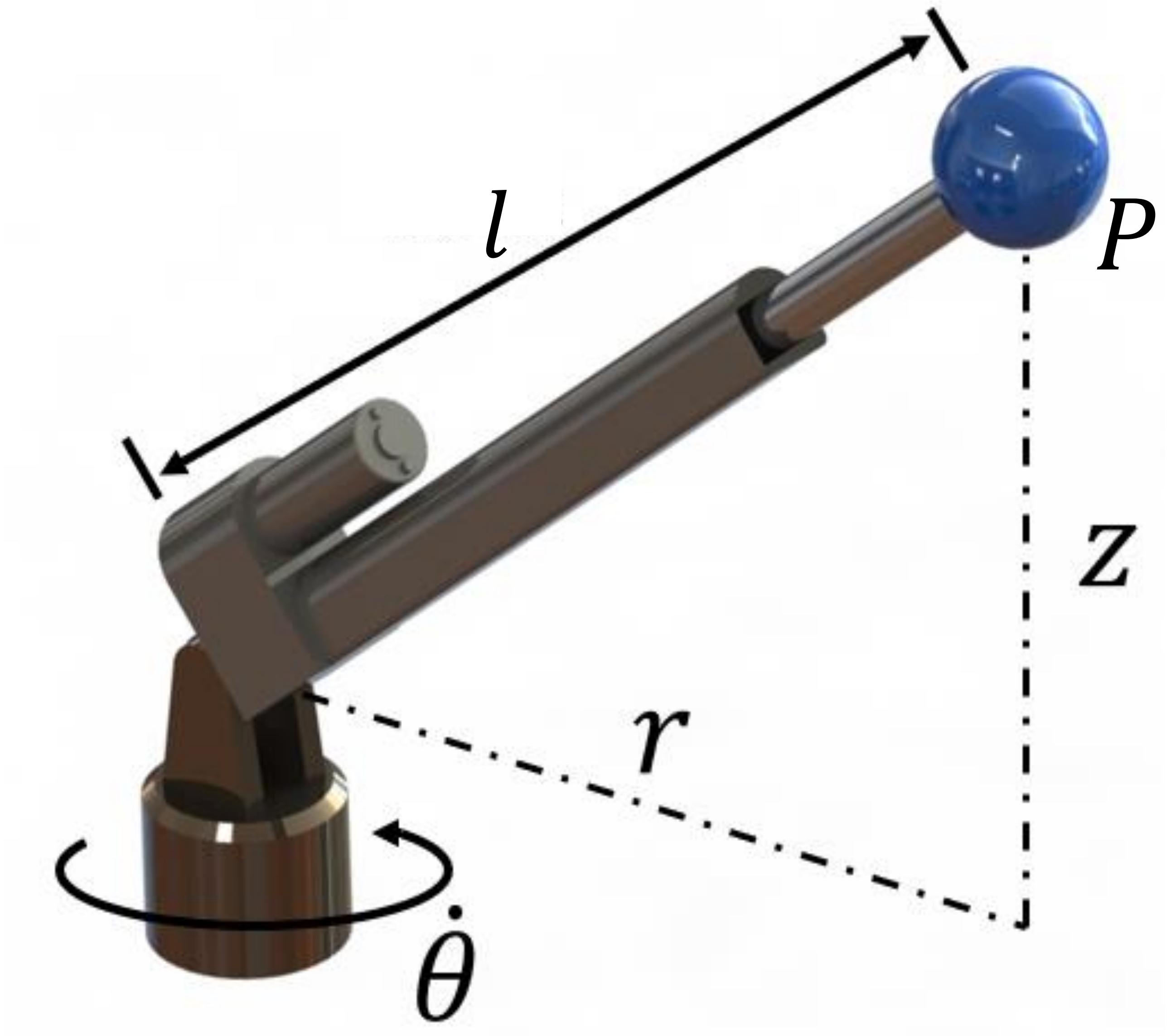
Question 4.10

A projectile is fired from the origin O in the x, y, z co-ordinate system with a speed of $35\,\text{m/s}$ in the direction shown in the diagram. Find the x, y and z components of position, velocity and acceleration 12 seconds after the projectile has been fired. Assume $g = 9.81\,m/s^2$ and air resistance as negligible.



Question 4.11

The linear actuator in question 4.7 is modified and then programmed to extend for a short time making $\dot{l} = 0.2\ \mathrm{m/s}$ when $l = 0.12\ \mathrm{m}$, and to simultaneously rotate according to $\theta = 0.8t$ radians. The height of the particle z varies according to $z = e^{-2t}\ \mathrm{m}$. If t is in seconds, find the magnitudes of velocity v and acceleration a of the point P on the end of length l when time $t = 2\ \mathrm{s}$.

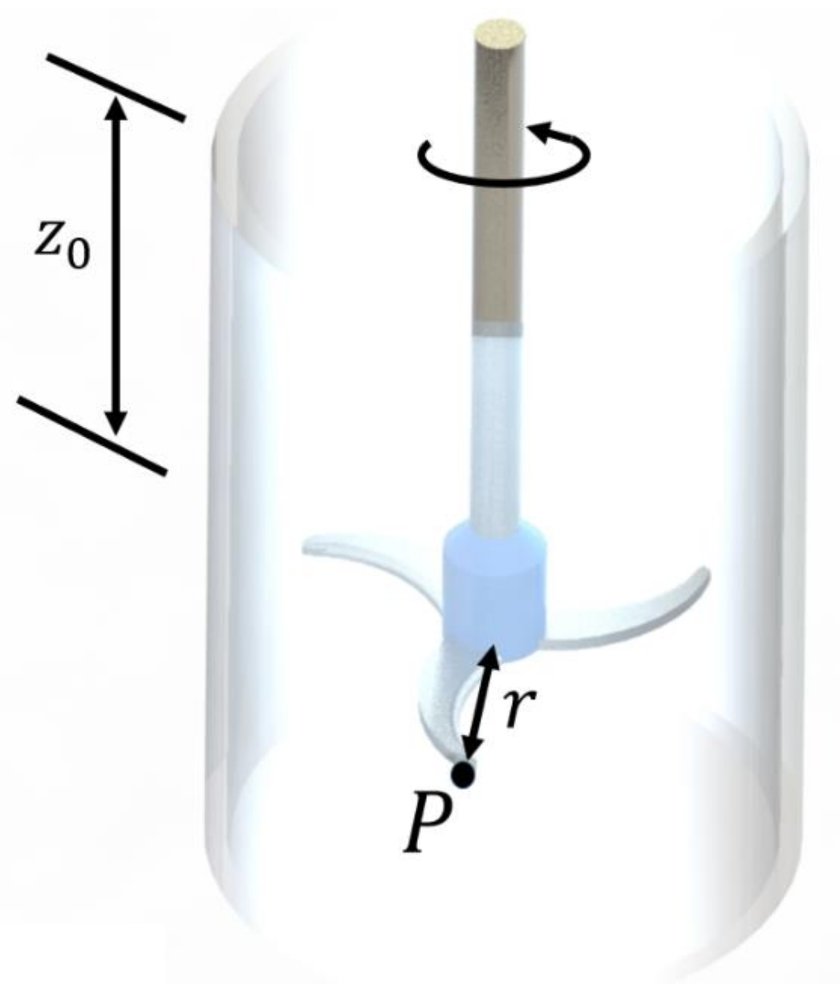


Question 4.12

The blade on an industrial food processor rotates at a constant rate of 2000 revolutions per minute, at the same time the blade moves in the vertical z direction according to $z = z_0 \cos 2\pi n t$ where the frequency $n = 0.5\ Hz$ is constant. Work out:

a) The magnitudes of velocity and acceleration of the point P on the blade when time $t = 2$ s.

b) The magnitude of the maximum possible acceleration of point P.



r = 0.5m

$z_0 = 0.8m$

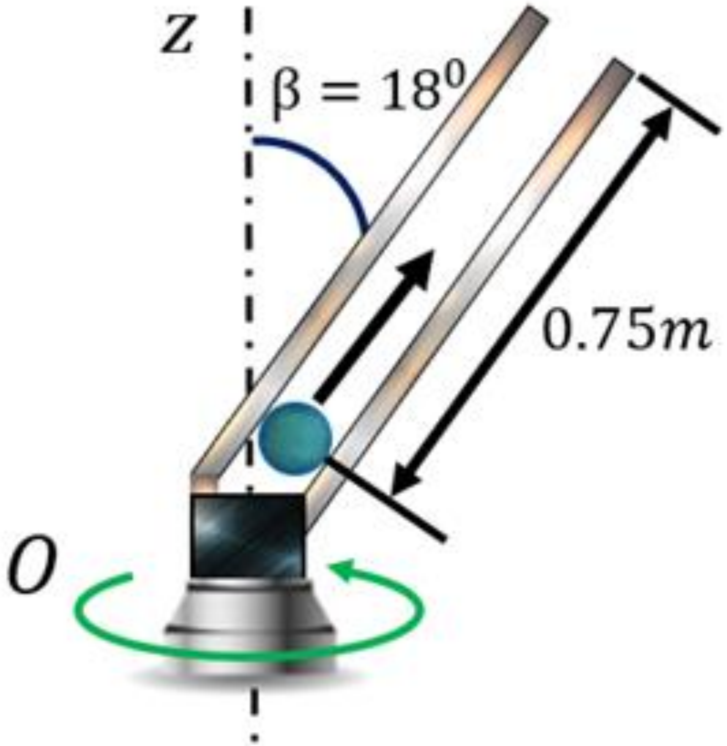
Question 4.13

The solid steel ball P rolls down a helical rail with a diameter of 35cm and lowering 2cm for every quarter of a turn. At the instant shown the ball P has just completed 3 quarters of its final full turn and has a speed of $3.5\,\mathrm{cm/s}$ which is increasing at a rate of $9\,\mathrm{cm/s^2}$. Find the components of velocity v and acceleration a in the u_r, u_θ and u_z directions.

Question 4.14

PEM tutoring

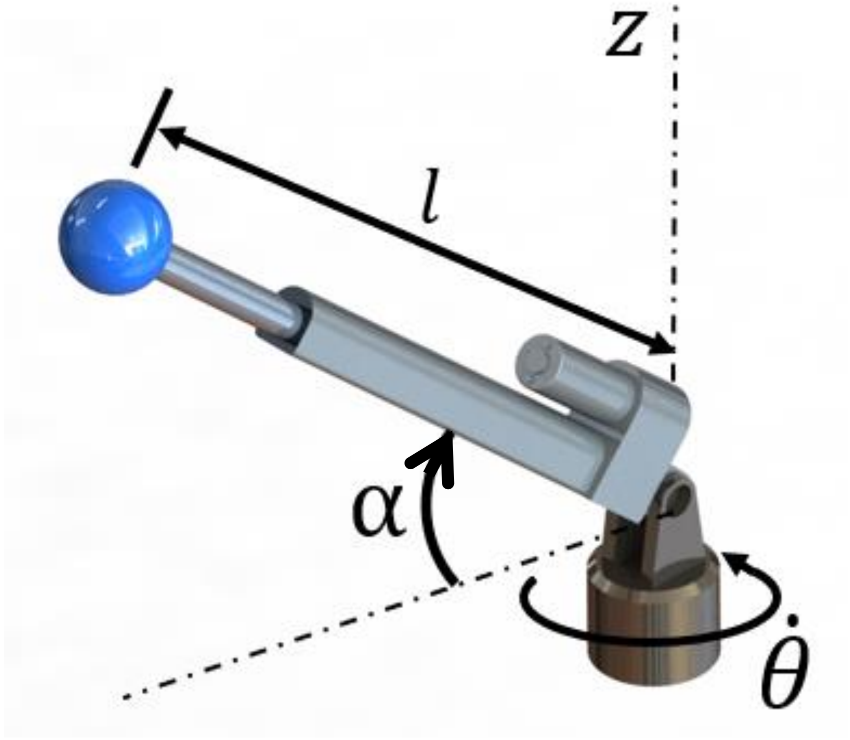
A particle is fired up a tube from rest with a constant acceleration of $6\,m/s^2$ and rotates at a constant rate of 5 revolutions per second. What are the magnitudes of velocity v and acceleration a of the particle at the point it leaves the tube.



Question 4.15

PEM tutoring

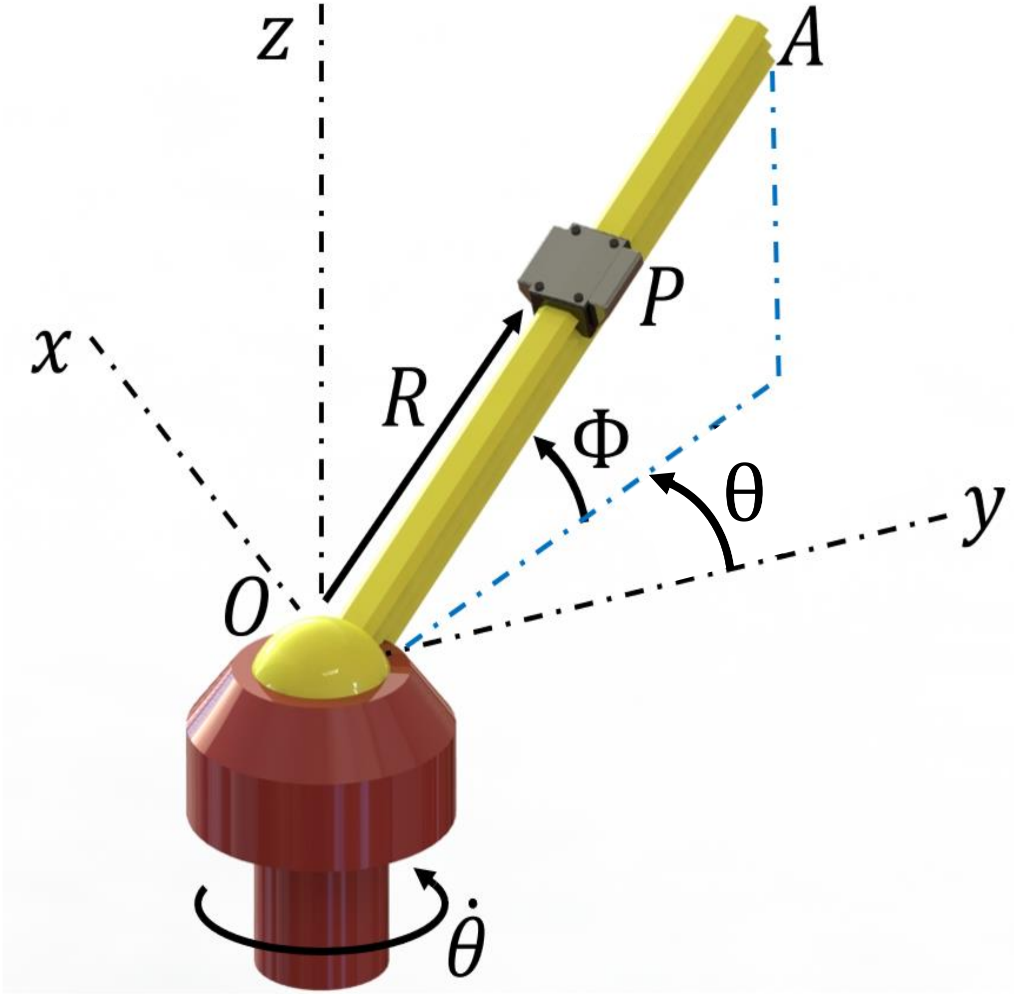
The linear actuator in question 4.11 is programmed to rotate about the z axis at a constant rate of $\dot{\theta} = 0.8\pi$ rads/s, the actuator can also rotate forming the angle α which changes at a constant rate of $\dot{\alpha} = 0.5\pi$ rads/s in the direction shown. If the arm extends at a constant rate of $\dot{l} = 0.2$ m/s, solve for a_R, a_θ and a_Φ at the instant $\alpha = 0.75$ radians and $l = 0.4$ m.



Question 4.16

PEM tutoring

The motorized slider at point P is able to move along the straight track OA according to $R = 0.5\sin\frac{\pi}{6}t$ meters, where t is in seconds. The track OA is attached to a ball joint at O which allows the track OA to be lifted from rest at a constant rate of $\dot{\Phi} = e^{\pi t}$ rads/s and rotate about the z axis at a constant rate of $\dot{\theta} = 10$ rads/s at the same time. Work out the magnitude of the total velocity and acceleration of the slider P when $t = 0.25$ s if $\theta = 0$ radians and $\Phi = \frac{\pi}{18}$ radians when $t = 0$ s.



Answers are shown on the next page

Answers

Question 4.1

a) $v = (12.5\ m/s)u_r + (6.55\ m/s)\ u_\theta$ $a = (-32.75\ m/s^2)u_r + (125\ m/s^2)u_\theta$
b) $v = 14.58\ m/s$ $a = 129.22\ m/s^2$

Question 4.2

a) $v = 40.13\ m/s$ $a = 576.91\ m/s^2$
b) $\theta_v = 169.02^0$ $\theta_a = 108.94^0$

Question 4.3

$v = 17.22\ m/s$

$a = 142.28\ m/s^2$

Question 4.4

differences due to rounding errors.

Answer 1: $v = 9.14\ m/s$ and $a = 47\ m/s^2$

Answer 2: $v = 9.13\ m/s$ and $a = 47.48\ m/s^2$

Question 4.5

a) $v = (45\ m/s)u_r + (23.58\ m/s)u_\theta$ $a = (-212.22\ m/s^2)u_r + (810\ m/s^2)u_\theta$
b) $v = (45\ m/s)u_r + (23.58\ m/s)u_\theta$ $a = (-197.22\ m/s^2)u_r + (817.86\ m/s^2)u_\theta$

Question 4.6

$v = (54\ m/s)u_r + (72\ m/s)u_\theta$

$a = (-105.05\ m/s^2)u_r + (-78.79\ m/s^2)u_\theta$

Question 4.7

$v = 0.21\ m/s \quad a = 0.3\ m/s^2$

Question 4.8

$v = 15\ m/s$

Question 4.9

differences due to rounding errors.

Using trigonometry

$v_r = 4.24\ m/s \qquad a_r = 1.06\ m/s^2$

$v_z = 6.8\ m/s \qquad a_z = 1.7\ m/s^2$

$v_\theta = 0\ m/s \qquad a_\theta = 0\ m/s^2$

Using vector dot product

$v_r = 4.25\ m/s \qquad a_r = 1.07\ m/s^2$

$v_z = 6.8\ m/s \qquad a_z = 1.7\ m/s^2$

$v_\theta = 0\ m/s \qquad a_\theta = 0\ m/s^2$

Question 4.10

$x = 117.66\ m \qquad v_x = 9.81\ m/s \qquad a_x = 0\ m/s^2$

$y = 82.39\ m \qquad v_y = 6.87\ m/s \qquad a_y = 0\ m/s^2$

$z = -311.64\ m \quad v_z = -84.83\ m/s \qquad a_z = 9.81\ m/s^2$

Question 4.11

$v = 0.22\ m/s \quad a = 0.34\ m/s^2$

Question 4.12

$v = 104.72\ m/s \quad a = 21932.56\ m/s^2$

Maximum acceleration occurs when $\cos 2\pi nt = 1$. So maximum acceleration is

$a_{max} = 21932.56\ m/s^2$

Question 4.13

$v_r = 0\ m/s \qquad a_r = 0.007\ m/s^2$

$v_z = -0.00258\ m/s \qquad a_z = 0.0066\ m/s^2$

$v_\theta = 0.0349\ m/s \qquad a_\theta = 0.0897\ m/s^2$

Question 4.14

$v = 7.87\ m/s \quad a = 232.74\ m/s^2$

Question 4.15

$a_r = -2.34\ m/s^2 \qquad a_\Phi = -0.63\ m/s^2 \quad a_\theta = 2.88\ m/s^2$

Question 4.16

$v = 0.62\ m/s \quad a = 7.83\ m/s^2$