Engineering Dynamics

Formula sheet

Rectilinear motion

$$\frac{ds}{dt} = v \qquad \frac{dv}{dt} = a \qquad ads = vdv$$

Constant acceleration:

$$v = v_0 + a_c t$$

$$s = s_0 + v_0 t + a_c \frac{t^2}{2}$$

$$v^2 = v_0^2 + 2a_c[s - s_0]$$

Acceleration as a function of time:

$$v = v_0 + \int_0^t f(t)\, dt$$

$$s = s_0 + \int_0^t [v_0 + \int_0^t f(t)\, dt]\, dt$$

Acceleration as a function of velocity:

$$\int_{v_0}^{v} \frac{dv}{f(v)} = \int_{t_0}^{t} dt$$

$$s = s_0 + \int_{v_0}^{v} \frac{v\, dv}{f(v)}$$

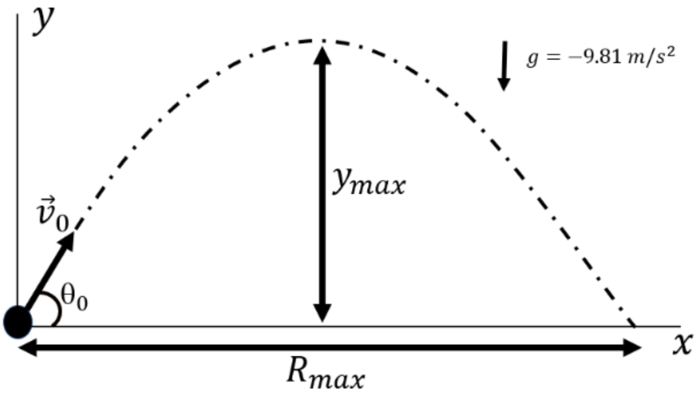
Acceleration as a function of position:

$$\int_{s_0}^{s} \frac{ds}{g(s)} = \int_{t_0}^{t} dt$$

$$v^2 = v_0^2 + 2\int_{s_0}^{s} f(s)\, ds$$

Projectile motion

Projectile launched from $y_0 = 0$ with angle θ_0



$$y_{max} = \frac{v_0^2\, sin^2(\theta_0)}{2g}$$

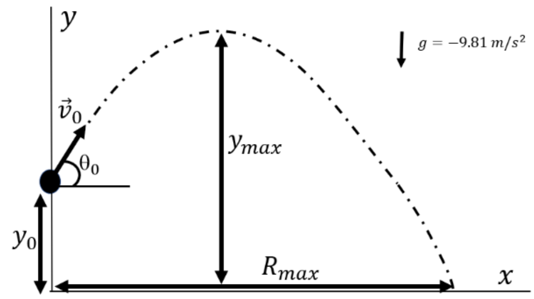
$$t = \frac{2v_0 \sin(\theta_0)}{g}$$

$$R_{max} = \frac{v_0^2}{g}\ \sin(2\theta_0)$$

$$v_{0x} = \vec{v}_0 \cos\theta_0$$

$$v_{0y} = \vec{v}_0 \sin\theta_0$$

Projectile launched from height y_0 with angle θ_0



$$y_{max} = \frac{v_0^2 \sin^2(\theta_0)}{2g} + y_0$$

$$t_{total} = \frac{v_0 \sin(\theta_0)}{g} + \sqrt{\frac{(v_0 \sin\theta_0)^2}{g^2} + \frac{2y_0}{g}}$$

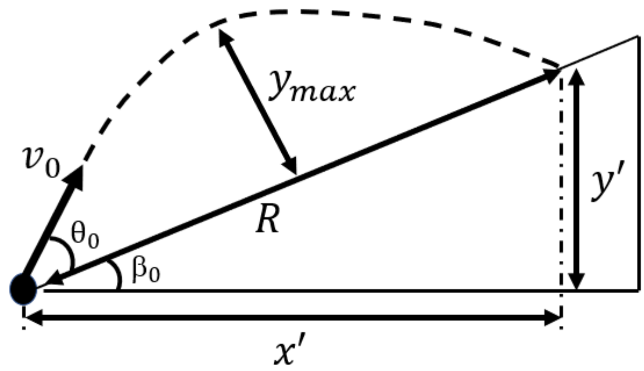
$$R_{max} = (v_0 \cos\theta_0)\left(\frac{v_0 \sin(\theta_0)}{g} + \sqrt{\frac{(v_0 \sin\theta_0)^2}{g^2} + \frac{2y_0}{g}}\right)$$

$$v_0 = \left(\frac{gt_{total}}{2} - \frac{y_0}{t_{total}}\right)\frac{1}{\sin\theta_0} = \left(\frac{gt_{total}}{2} - \frac{y_0}{t_{total}}\right)\csc\theta_0$$

Projectile launched up an inclined plane with angle β_0

$$t = \frac{2v_0 \sin(\theta_0)}{g \cos\beta_0}$$

$$x' = \frac{2{v_0}^2(\cos\beta_0 + \theta_0)(\sin\theta_0)}{g\cos\beta_0}$$



$$R = \frac{2{v_0}^2(\cos\beta_0 + \theta_0)(\sin\theta_0)}{g\cos^2\beta_0}$$

$$y' = \frac{2{v_0}^2(\cos\beta_0 + \theta_0)(\sin\theta_0)(\sin\beta_0)}{g\cos^2\beta_0}$$

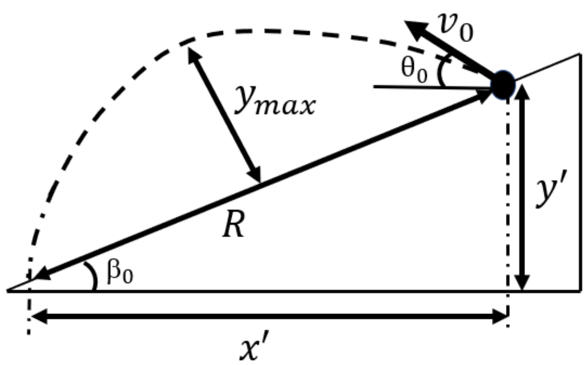
$$y_{max} = \frac{(v_0\sin\theta_0)^2}{2g\cos\beta_0}$$

Angle for maximum range: $\theta_0 = \frac{90+\beta_0}{2} - \beta_0$

Projectile launched down an inclined plane with angle β_0

$$t = \frac{2v_0\sin\beta_0 + \theta_0}{g\cos\beta_0}$$

$$x' = \frac{2{v_0}^2(\cos\theta_0)(\sin\beta_0 + \theta_0)}{g\cos\beta_0}$$



$$R = \frac{2{v_0}^2(\cos\theta_0)(\sin\beta_0 + \theta_0)}{g\cos^2\beta_0}$$

$$y' = \frac{2{v_0}^2(\cos\theta_0)(\sin\beta_0 + \theta_0)(\sin\beta_0)}{g\cos^2\beta_0}$$

$$y_{max} = \frac{\left(v_0\sin\theta_0 + \beta_0\right)^2}{2g\cos\beta_0}$$

Angle for maximum range: $\theta_0 = \frac{90+\beta_0}{2}$

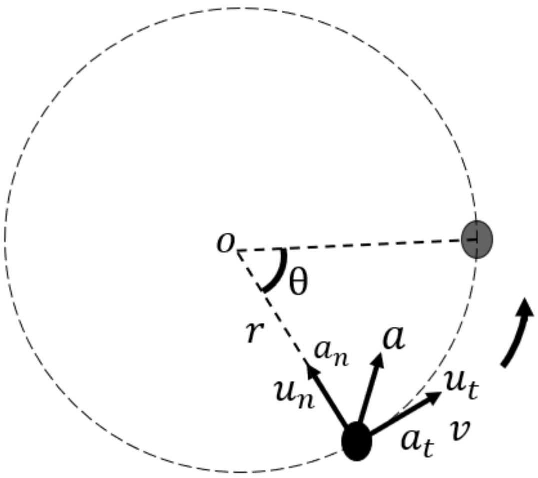
Normal – tangential co-ordinates

$$\theta = \dot{\theta} t \qquad s = r\theta \qquad v = r\dot{\theta} \qquad a_t = r\ddot{\theta} \qquad a_n = \frac{v^2}{r} = \dot{\theta}^2 r$$

$$a = \sqrt{{a_n}^2 + {a_t}^2} \qquad \dot{\theta} = \dot{\theta}_0 + \ddot{\theta} t \qquad \theta = \theta_0 + \dot{\theta}_0 t + \ddot{\theta}\frac{t^2}{2}$$

$$\dot{\theta}^2 = {\dot{\theta}_0}^2 + 2\ddot{\theta}[\theta - \theta_0]$$

$$\rho = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$



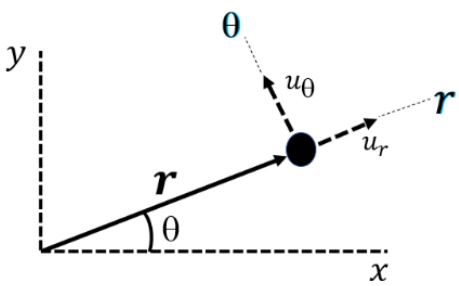
Polar co-ordinates

$$\boldsymbol{r} = r u_r \qquad v = \dot{r} u_r + r\dot{\theta} u_\theta \qquad v_r = \dot{r} \qquad v_\theta = r\dot{\theta}$$

$$a = \left(\ddot{r} - r\dot{\theta}^2\right) u_r + (2\dot{\theta}\dot{r} + r\ddot{\theta}) u_\theta \qquad a_r = \ddot{r} - r\dot{\theta}^2 \qquad a_\theta = 2\dot{\theta}\dot{r} + r\ddot{\theta}$$

$$a_\theta = \frac{1}{r}\frac{d}{dt}\left(r^2\dot{\theta}\right)$$

$$v = \sqrt{{v_r}^2 + {v_\theta}^2} \qquad a = \sqrt{{a_r}^2 + {a_\theta}^2}$$



Cylindircal co-ordinates

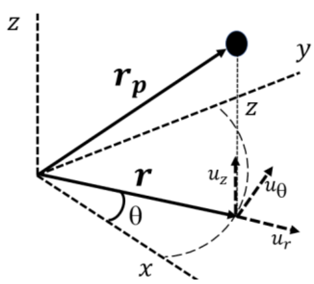
$$\boldsymbol{r_p} = r u_r + z u_z$$

$$v = \dot{r} u_r + r\dot{\theta} u_\theta + \dot{z} u_z \qquad v_r = \dot{r} \qquad v_\theta = r\dot{\theta} \qquad v_z = \dot{z}$$

$$a = \left(\ddot{r} - r\dot{\theta}^2\right) u_r + (2\dot{\theta}\dot{r} + r\ddot{\theta}) u_\theta + (\ddot{z}) u_z$$

$$a_r = \ddot{r} - r\dot{\theta}^2 \qquad a_\theta = 2\dot{\theta}\dot{r} + r\ddot{\theta} \qquad a_z = \ddot{z}$$

$$v = \sqrt{{v_r}^2 + {v_\theta}^2 + {v_z}^2} \qquad a = \sqrt{{a_r}^2 + {a_\theta}^2 + {a_z}^2}$$



Spherical co-ordinates

$$\dot{R} = \dot{R}u_R + R\dot{\Phi}u_\Phi + R\dot{\theta}\sin\Phi\, u_\theta$$

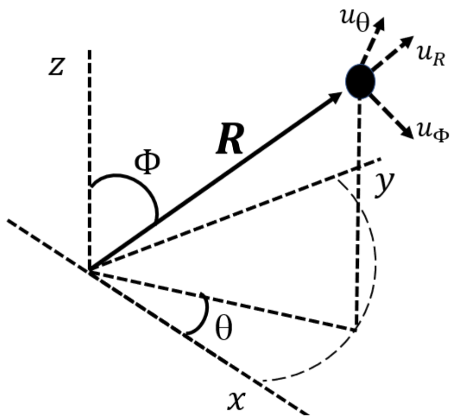
$$v_R = \dot{R} \quad v_\Phi = R\dot{\Phi} \quad v_\theta = R\dot{\theta}\sin\Phi$$

$$\ddot{R} = \left(\ddot{R} - R\dot{\Phi}^2 - R\dot{\theta}^2\sin^2\Phi\right)u_R + \left(2\dot{R}\dot{\Phi} + R\ddot{\Phi} - R\dot{\theta}^2\sin\Phi\cos\Phi\right)u_\Phi + \left(2\dot{R}\dot{\theta}\sin\Phi + 2R\dot{\Phi}\dot{\theta}\cos\Phi + R\ddot{\theta}\sin\Phi\right)u_\theta$$

$$a_R = \ddot{R} - R\dot{\Phi}^2 - R\,\dot{\theta}^2\sin^2\Phi$$

$$a_\Phi = \frac{1}{R}\left(\frac{d}{dt}\left(R^2\dot{\Phi}\right)\right) - R\,\dot{\theta}^2 \sin\Phi\cos\Phi$$

$$a_\theta = \frac{\sin\Phi}{R}\left(\frac{d}{dt}\left(R^2\dot{\theta}\right)\right) + 2R\dot{\Phi}\dot{\theta}\cos\Phi$$



Relative motion

$$r_B = r_A + r_{B/A}$$

$$x_2 i + y_2 j = (x_1 i + y_1 j) + (x' i + y' j)$$

$$V_B = V_A + V_{B/A}$$

$$v_B i + v_B j = (v_A i + v_A j) + \left(v_{B/A} i + v_{B/A} j\right)$$

$$a_B = a_A + a_{B/A}$$

$$a_B i + a_B j = (a_A i + a_A j) + \left(a_{B/A} i + a_{B/A} j\right)$$

Newtons second law

$$\sum \vec{F}_x\, i + \sum \vec{F}_y\, j + \sum \vec{F}_z\, k = m\left(\vec{a}_x i + \vec{a}_y j + \vec{a}_z k\right)$$

$$\sum \vec{F} = \left(\sum \vec{F}_x\, i\right) + \left(\sum \vec{F}_y\, j\right) + \left(\sum \vec{F}_z\, k\right)$$

$$\left|\sum \vec{F}\right| = \sqrt{\left(\sum \vec{F}_x\, i\right)^2 + \left(\sum \vec{F}_y\, j\right)^2 + \left(\sum \vec{F}_z\, k\right)^2}$$

$$\vec{a} = \vec{a}_x i + \vec{a}_y j + \vec{a}_z k$$

$$|\vec{a}| = \sqrt{(\vec{a}_x i)^2 + \left(\vec{a}_y j\right)^2 + (\vec{a}_z k)^2}$$

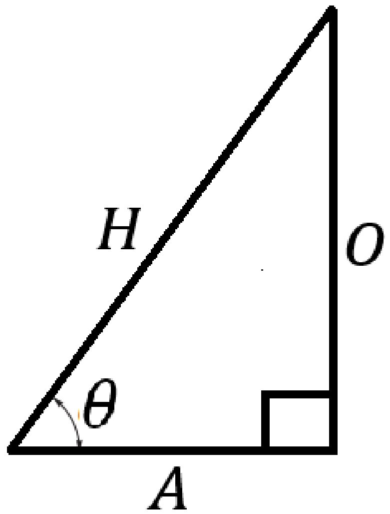
$$\left|\sum \vec{F}\right| = m|\vec{a}|$$

Trigonometric identities

$\sin\theta = \frac{O}{H} \quad \csc\theta = \frac{H}{O}$

$\cos\theta = \frac{A}{H} \quad \sec\theta = \frac{H}{A}$

$\tan\theta = \frac{O}{A} \quad \cot\theta = \frac{A}{O}$



$$\sin^2(\theta) - \cos^2(\theta) = 1$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin(\theta \pm \beta) = \cos(\theta)\cos(\beta) \pm \sin(\beta)\sin(\theta)$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\sin(\theta) = \pm\sqrt{\frac{1-\cos(2\theta)}{2}}$$

$$\cos(\theta) = \pm\sqrt{\frac{1+\cos(2\theta)}{2}}$$

$$1 + \tan^2(\theta) = \sec^2(\theta)$$

$$1 + \cot^2(\theta) = \csc^2(\theta)$$

Derivatives

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$ Product rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$ Quotient rule

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\sec x) = \tan x \sec x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\cosh x) = \sinh x$$

Chain rule (where u is a function of x):

$$\frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx}$$

$$\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$$

$$\frac{d}{dx}(\csc u) = -\csc u \cot u \frac{du}{dx}$$

$$\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx}(\sec u) = \tan u \sec u \frac{du}{dx}$$

$$\frac{d}{dx}(\tan u) = \sec^2 u \frac{du}{dx}$$

$$\frac{d}{dx}(\cot u) = -\csc^2 u \frac{du}{dx}$$

$$\frac{d}{dx}(\sinh u) = \cosh u \frac{du}{dx}$$

$$\frac{d}{dx}(\cosh u) = \sinh u \frac{du}{dx}$$

Intergrals

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}$$

$$\int \frac{1}{x} \, dx = \ln x$$

$$\int \sin x \;\, dx = -\cos x$$

$$\int \cos x \;\, dx = \sin x$$

$$\int \sec x \;\, dx = \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x}$$

$$\int \sin^2 x \;\, dx = \frac{x}{2} - \frac{\sin 2x}{4}$$

$$\int \cos^2 x \;\, dx = \frac{x}{2} + \frac{\sin 2x}{4}$$

$$\int \sin x \cos x \; dx = \frac{\sin^2 x}{2}$$

$$\int \sinh x \; dx = \cosh x$$

$$\int \cosh x \; dx = \sinh x$$

$$\int \tanh x \; dx = \ln \cosh x$$

$$\int \ln x \; dx = x \ln x - x$$

$$\int x \; \sin x \; dx = \sin x - x \cos x$$

$$\int x \cos x \; dx = \cos x + x \sin x$$

$$\int x^2 \sin x \; dx = 2x \sin x - (x^2 - 2) \cos x$$

$$\int x^2 \cos x \; dx = 2x \cos x + (x^2 - 2) \sin x$$

$$\int e^{ax} \, dx = \frac{e^{ax}}{a}$$

$$\int x \, e^{ax} \, dx = \frac{e^{ax}}{a^2}(ax - 1)$$

$$\int e^{ax} \sin bx \; dx = \frac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2}$$

$$\int e^{ax} \cos bx \; dx = \frac{e^{ax}(a \cos bx + b \sin bx)}{a^2 + b^2}$$

$$\int e^{ax}\sin^2 x\ dx = \frac{e^{ax}}{4+a^2}\left(a\sin^2 x - \sin 2x + \frac{2}{a}\right)$$

$$\int e^{ax}\cos^2 x\ dx = \frac{e^{ax}}{4+a^2}\left(a\cos^2 x + \sin 2x + \frac{2}{a}\right)$$

$$\int e^{ax}\sin x\cos x\ dx = \frac{e^{ax}}{4+a^2}\left(\frac{a}{2}\sin 2x - \cos 2x\right)$$

$$\int \sqrt{a+bx}\ dx = \frac{2}{3b}\sqrt{(a+bx)^3}$$

$$\int x\sqrt{a+bx}\ dx = \frac{2}{15b^2}\ (3bx-2a)\sqrt{(a+bx)^3}$$

$$\int x^2\sqrt{a+bx}\ dx = \frac{2}{105b^3}(8a^2-12abx+15b^2x^2)\sqrt{(a+bx)^3}$$

$$\int \frac{dx}{\sqrt{a+bx}} = \frac{2\sqrt{a+bx}}{b}$$

$$\int \frac{x\,dx}{a+bx} = \frac{1}{b^2}[a+bx-a\ln(a+bx)]$$

$$\int \frac{x\,dx}{(a+bx)^n} = \frac{(a+bx)^{1-n}}{b^2}\left[\frac{a+bx}{2-n}-\frac{a}{1-n}\right]$$

$$\int \frac{dx}{a+bx^2} = \frac{1}{\sqrt{ab}}\tan^{-1}\frac{x\sqrt{ab}}{a}$$

$$\int \frac{x\,dx}{a+bx^2} = \frac{1}{2b}\ln(a+bx^2)$$

$$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\sin^{-1}\frac{x}{a}\right]$$

$$\int x\sqrt{a^2-x^2}\,dx = -\frac{1}{3}\sqrt{(a^2-x^2)^3}$$

$$\int x^2\sqrt{a^2-x^2}\,dx = -\frac{x}{4}\sqrt{(a^2-x^2)^3}+\frac{a^2}{8}\left(x\sqrt{a^2-x^2}+a^2\sin^{-1}\frac{x}{a}\right)$$

$$\int \frac{dx}{\sqrt{a+bx+cx^2}} = -\frac{1}{\sqrt{c}}\ln\left(\sqrt{a+bx+cx^2}+x\sqrt{c}+\frac{b}{2\sqrt{c}}\right)$$

$$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left(x+\sqrt{x^2\pm a^2}\right)$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$$

$$\int \frac{x\,dx}{\sqrt{x^2-a^2}} = \sqrt{x^2-a^2}$$

$$\int \frac{x\,dx}{\sqrt{a^2\pm x^2}} = \pm\sqrt{a^2\pm x^2}$$

$$\int x\sqrt{x^2\pm a^2}\,dx = \frac{1}{3}\sqrt{(x^2\pm a^2)^3}$$

$$\int x^2\sqrt{x^2\pm a^2}\,dx = \frac{x}{4}\sqrt{(x^2\pm a^2)^3}\pm\frac{a^2}{8}x\sqrt{(x^2\pm a^2)}-\frac{a^4}{8}\ln\left(x+\sqrt{(x^2\pm a^2)}\right)$$